Review for Chapter 4 Test #2

Name__________________

Directions: For each problem, write an equation that will relate the information in the problem. Use derivatives to solve the problem then provide the answer to the problem in a complete sentence.

Solve the problem.

1) The radius of a right circular cylinder is increasing at the rate of 5 in./s, while the height is decreasing at the rate of 8 in./s. At what rate is the volume of the cylinder changing when the radius is 15 in. and the height is 20 in.?

2) A spherical balloon is inflated with helium at a rate of 90π ft³/min. How fast is the balloon's radius increasing when the radius is 3 ft?

3) Water is falling on a surface, wetting a circular area that is expanding at a rate of 8 mm²/s. How fast is the radius of the wetted area expanding when the radius is 188 mm? (Round approximations to four decimal places.)

4) A man flies a kite at a height of 50 m. The wind carries the kite horizontally away from him at a rate of 10 m/sec. How fast is the distance between the man and the kite changing when the kite is 130 m away from him?

5) A rectangular swimming pool 16 m by 11 m is being filled at the rate of 0.4 m³/min. How fast is the height h of the water rising?

6) A ladder is slipping down a vertical wall. If the ladder is 10 ft long and the top of it is slipping at the constant rate of 3 ft/s, how fast is the bottom of the ladder moving along the ground when the bottom is 8 ft from the wall?

7) A metal cube dissolves in acid such that an edge of the cube decreases by 0.58 mm/min. How fast is the volume of the cube changing when the edge is 7.5 mm?

8) One airplane is approaching an airport from the north at 181 km/hr. A second airplane approaches from the east at 191 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 34 km away from the airport and the westbound plane is 21 km from the airport.
9) A piece of land is shaped like a right triangle. Two people start at the right angle at the same time, and walk at the same speed along different legs of the triangle while spraying the land. If the area covered is changing at 4 m²/s, how fast are the people moving when they are 3 m from the right angle? (Round approximations to two decimal places.)

10) A carpenter is building a rectangular room with a fixed perimeter of 120 ft. What are the dimensions of the largest room that can be built? What is its area?

11) Find the dimensions that produce the maximum floor area for a one-story house that is rectangular in shape and has a perimeter of 149 ft. Round to the nearest hundredth, if necessary.

12) An architect needs to design a rectangular room with an area of 75 ft². What dimensions should he use in order to minimize the perimeter? Round to the nearest hundredth, if necessary.

13) A company wishes to manufacture a box with a volume of 24 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material. Round to the nearest tenth, if necessary.

14) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 42.5 ft³. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

15) If the price charged for a bolt is p cents, then x thousand bolts will be sold in a certain hardware store, where \( p = 71 - \frac{x}{34} \). How many bolts must be sold to maximize revenue?
16) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 96 in. What dimensions will give a box with a square end the largest possible volume?

17) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:
   \[ R(x) = 70x - 0.5x^2 \]
   \[ C(x) = 4x + 2. \]

18) A trough is to be made with an end of the dimensions shown. The length of the trough is to be 17 feet long. Only the angle \( \theta \) can be varied. What value of \( \theta \) will maximize the trough's volume?
1) $1200\pi$ in.$^3$/s
2) 2.50 ft/min
3) 0.0068 mm/s
4) 9.2 m/sec
5) 0.0023 m/min
6) 2.3 ft/s
7) $-98$ mm$^3$/min
8) 380 km/hr
9) 1.33 m/s
10) 30 ft $\times$ 30 ft; 900 ft$^2$
11) 37.25 ft $\times$ 37.25 ft
12) 8.66 ft $\times$ 8.66 ft
13) 2.6 ft
14) 4.4 ft $\times$ 4.4 ft. $\times$ 2.2 ft
15) 1207 thousand bolts
16) 16 in. $\times$ 16 in. $\times$ 32 in.
17) 66 units
18) $30^\circ$